

# Integrable inhomogeneous Lakshmanan-Myrzakulov equation

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**Abstract.** The integrable inhomogeneous extension of the Lakshmanan-Myrzakulov equation is constructed by using the prolongation structure theory. The corresponding L-equivalent counterpart is also given, which is the (2+1)-dimensional generalized NLSE.

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## 1 Introduction

As an important subclass of integrable nonlinear differential equations, the Heisenberg ferromagnet equation (HFE),  $\mathbf{S}_t = \mathbf{S} \times \mathbf{S}_{xx}$ , and its (2+1)-dimensional integrable extensions have been paid more attention [1-10]. Recently, a new (2+1)-dimensional integrable extension was introduced by Lakshmanan and Myrzakulov (see, e.g. Ref. [11]),

$$\begin{aligned}\mathbf{S}_t &= \{\mathbf{S} \times (\alpha \mathbf{S}_x + \beta \mathbf{S}_y) + u \mathbf{S}\}_x, \\ u_x &= -\beta \mathbf{S} \cdot (\mathbf{S}_x \times \mathbf{S}_y),\end{aligned}\tag{1}$$

where  $\mathbf{S} = (S_1, S_2, S_3)$  is the spin vector,  $\varepsilon S_1^2 + \varepsilon S_2^2 + S_3^2 = 1$  (for compact case  $\varepsilon = 1$  and for noncompact case  $\varepsilon = -1$ ),  $u$  is the real scalar function,  $\alpha, \beta$  are real constants. When  $\alpha = 0, \beta = 1$ , Eq.(1) reduces to the Myrzakulov-I (M-I) equation (see, e.g. Refs. [5-10]),

$$\begin{aligned}\mathbf{S}_t &= \{\mathbf{S} \times \mathbf{S}_y + u \mathbf{S}\}_x, \\ u_x &= -\mathbf{S} \cdot (\mathbf{S}_x \times \mathbf{S}_y).\end{aligned}\tag{2}$$

In Ref.[12], it was shown that Lakshmanan-Myrzakulov equation (LME) (1) is L-equivalent (about our conditional notations, see e.g. Refs. [9-10]) and gauge equivalent to the following integrable (2+1)-dimensional nonlinear Schrödinger equation (NLSE)

$$\begin{aligned}iq_t - \alpha q_{xx} - \beta q_{xy} - vq &= 0, \\ ip_t + \alpha p_{xx} + \beta p_{xy} + vp &= 0, \\ v_x &= 2[\alpha(pq)_x + \beta(pq)_y],\end{aligned}\tag{3}$$

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where  $p = \varepsilon \bar{q}$ .

As known, some integrable systems admit integrable inhomogeneous extensions [13-14]. More recently, the integrable inhomogeneous Myrzakulov-I (M-I) equation has been constructed in Ref.[14]. Note that M-I equation is just a special case of (1). Thus an interesting question that naturally arises is whether there is more general integrable inhomogeneous extension of (1). The aim of this paper is to construct such integrable inhomogeneous extension of (1) and explore some of its properties.

## 2 The integrable inhomogeneous LME

Let us consider the following inhomogeneous LME,

$$\begin{aligned}\mathbf{S}_t &= \{\mathbf{S} \times (f_1 \mathbf{S}_x + \beta \mathbf{S}_y) + u \mathbf{S}\}_x + f_2 \mathbf{S}_x, \\ u_x &= -\beta \mathbf{S} \cdot (\mathbf{S}_x \times \mathbf{S}_y),\end{aligned}\tag{4}$$

where  $\beta$  should be the constant as in Ref.[14],  $f_1$  and  $f_2$  which are the functions of  $(x, t)$  need to be determined. In order to construct the integrable inhomogeneous LME, we shall analyze Eq.(4) by using the prolongation structure theory as done in Ref.[15]. Let us first consider the prolongation structure of (4) for the case of  $\mathbf{S}_t = 0$ . Setting  $\mathbf{W} = \mathbf{S}_x$ ,  $\mathbf{T} = \mathbf{S}_y$  and taking  $\mathbf{S}, \mathbf{T}, \mathbf{W}$ , and  $u$  as the new independent variables, we can define the following set of two forms,

$$\begin{aligned}\alpha_a &= dS_a \wedge dx - T_a dy \wedge dx, \\ \alpha_{a+3} &= dS_a \wedge dy - W_a dx \wedge dy, \\ \alpha_{a+6} &= \beta(\mathbf{W} \times \mathbf{T})_a dx \wedge dy + \beta(\mathbf{S} \times d\mathbf{T})_a \wedge dy + S_a du \wedge dy \\ &\quad + u W_a dx \wedge dy + f_1(\mathbf{S} \times d\mathbf{W})_a \wedge dy + f_{1x}(\mathbf{S} \times \mathbf{W})_a dx \wedge dy + f_2(\mathbf{W})_a dx \wedge dy, \\ \alpha_{10} &= du \wedge dy + \beta \mathbf{S} \cdot (\mathbf{W} \times \mathbf{T}) dx \wedge dy, \\ \alpha_{a+10} &= dT_a \wedge dy + dW_a \wedge dx, \\ \alpha_{14} &= S_a dW_a \wedge dy + W_a dS_a \wedge dy, \\ \alpha_{15} &= (\mathbf{T} \cdot \mathbf{W}) dx \wedge dy + S_a \cdot dT_a \wedge dy,\end{aligned}\tag{5}$$

where  $a = 1, 2, 3$ , such that they constitute a closed ideal  $I = \{\alpha_i, i = 1, 2, \dots, 15\}$ . Then we extend the above ideal I by adding to it a set of one forms,

$$\Omega^k = d\xi^k + F^k(x, y, \mathbf{S}, \mathbf{T}, \mathbf{W}, u) \xi^k dx + G^k(x, y, \mathbf{S}, \mathbf{T}, \mathbf{W}, u) \xi^k dy, \quad k = 1, 2, \dots, n,\tag{6}$$

where  $\xi^k$  is prolongation variable. In terms of the prolongation condition,  $d\Omega^k \subset \{I, \Omega^k\}$ , we obtain the following set of partial differential equations for  $F^k$  and  $G^k$ ,

$$\begin{aligned} \frac{\partial F^k}{\partial T_a} &= \frac{\partial F^k}{\partial u} = 0, \quad \frac{\partial G^k}{\partial W_a} - \frac{f_1}{\beta} \frac{\partial G^k}{\partial T_a} = 0, \\ - \frac{\partial F^k}{\partial S_a} T_a + \frac{\partial G^k}{\partial S_a} W_a - \frac{\partial G^k}{\partial u} \beta \mathbf{S} \cdot (\mathbf{W} \times \mathbf{T}) + \left( \frac{\partial G^k}{\partial T_a} - \frac{\partial F^k}{\partial W_a} \right) \{ [\mathbf{S} \times (\mathbf{W} \times \mathbf{T})]_a \\ - S_a (\mathbf{T} \cdot \mathbf{W}) + \frac{u}{\beta} (\mathbf{S} \times \mathbf{W})_a - \frac{f_1}{\beta} (\mathbf{W} \cdot \mathbf{W}) S_a - \frac{f_{1x}}{\beta} W_a \} - [F, G]^k + \frac{\partial G^k}{\partial x} - \frac{\partial F^k}{\partial y} &= 0, \end{aligned} \quad (7)$$

where  $[F, G]^k \equiv \sum_{l=1}^n F^l \frac{\partial G^k}{\partial y^l} - \sum_{l=1}^n G^l \frac{\partial F^k}{\partial y^l}$ . By solving (7), we have the following solution,

$$F = \lambda \sum_{i=1}^3 S_i X_i, \quad G = \left( -\lambda \frac{f_1}{\beta} + \frac{f_2}{\beta} + \frac{u}{\beta} \right) \sum_{i=1}^3 S_i X_i + \frac{f_1}{\beta} \sum_{i=1}^3 (\mathbf{S} \times \mathbf{W})_i X_i + \sum_{i=1}^3 (\mathbf{S} \times \mathbf{T})_i X_i, \quad (8)$$

where  $X_i$ ,  $i = 1, 2, 3$ , depend only on the prolongation variables  $\xi^k$  and have the commutation relation of the  $su(2)$  Lie algebra.

Now let us define a set of 3-form  $\bar{\alpha}_i$  as follows,

$$\begin{aligned} \bar{\alpha}_a &= dS_a \wedge dx \wedge dt - T_a dy \wedge dx \wedge dt, \\ \bar{\alpha}_{a+3} &= dS_a \wedge dy \wedge dt - W_a dx \wedge dy \wedge dt, \\ \bar{\alpha}_{a+6} &= \beta (\mathbf{W} \times \mathbf{T})_a dx \wedge dy \wedge dt + \beta (\mathbf{S} \times d\mathbf{T})_a \wedge dy \wedge dt + S_a du \wedge dy \wedge dt + u W_a dx \wedge dy \wedge dt \\ &\quad + f_1 (\mathbf{S} \times d\mathbf{W})_a \wedge dy \wedge dt + f_{1x} (\mathbf{S} \times \mathbf{W})_a dx \wedge dy \wedge dt + f_2 (\mathbf{W})_a dx \wedge dy \wedge dt - dS_a \wedge dx \wedge dy, \\ \bar{\alpha}_{10} &= du \wedge dy \wedge dt + \beta \mathbf{S} \cdot (\mathbf{W} \times \mathbf{T}) dx \wedge dy \wedge dt, \\ \bar{\alpha}_{a+10} &= dT_a \wedge dy \wedge dt + dW_a \wedge dx \wedge dt, \\ \bar{\alpha}_{14} &= S_a dW_a \wedge dy \wedge dt + W_a dS_a \wedge dy \wedge dt, \\ \bar{\alpha}_{15} &= (\mathbf{T} \cdot \mathbf{W}) dx \wedge dy \wedge dt + S_a \cdot dT_a \wedge dy \wedge dt, \end{aligned} \quad (9)$$

where  $a = 1, 2, 3$ , such that they constitute a closed ideal. When these two forms are null, we recover (4). Then we introduce the following two forms,

$$\bar{\Omega}^k = \Omega^k \wedge dt + H_j^k \xi^j dx \wedge dy + (A_j^k dx + B_j^k dy) \wedge d\xi^j, \quad k = 1, 2, \dots, n, \quad (10)$$

where the matrices of A and B depend on the variables (x, y, t) and the form of  $\Omega^k$  is given by (6), in which  $\lambda$  depends on the variables (x, y, t). It is easily shown that

$$d\bar{\Omega}^k = \sum_{i=1}^{15} g^{ki} \bar{\alpha}_i + \sum_{j=1}^n \zeta_j^k \wedge \bar{\Omega}^j, \quad (11)$$

provided that the matrix H is given by

$$H = GA - FB + A_y - B_x + A_t B - B_t A \quad (12)$$

and

$$\begin{aligned} dH \wedge dx \wedge dy - \frac{1}{\beta} \frac{\partial G}{\partial T_a} (\mathbf{S} \times \mathbf{dS})_a \wedge dx \wedge dy - \lambda_y S_a X_a dx \wedge dy \wedge dt \\ (-\frac{(\lambda f_1)_x}{\beta} + \frac{f_{2x}}{\beta}) S_a X_a dx \wedge dy \wedge dt - A_t G dx \wedge dy \wedge dt + B_t F dx \wedge dy \wedge dt = 0. \end{aligned} \quad (13)$$

Substituting the expressions (8) of F and G into (12) and (13), we obtain

$$A = 0, \quad B = \frac{1}{\beta \lambda} I, \quad (14)$$

and

$$\lambda_t = -\beta \lambda \lambda_y - \lambda^2 f_{1x} + \lambda f_{2x}, \quad \lambda_x = 0. \quad (15)$$

From Eq.(15), we find that  $f_i$ ,  $i = 1, 2$ , should take the following expressions

$$f_i = \mu_i(t)x + \nu_i(t). \quad (16)$$

By restricting (10) on the solution manifold, we obtain the Lax representation of equation (4)

$$\begin{aligned} \xi_x &= -F|_{X_i = -\frac{i}{2}\sigma_i} \xi = \frac{i\lambda}{2} \sum_{i=1}^3 S_i \sigma_i \xi, \\ \xi_t &= -\frac{1}{B} \xi_y - \frac{1}{B} G|_{X_i = -\frac{i}{2}\sigma_i} \xi \\ &= -\beta \lambda \xi_y + \frac{i\lambda}{2} \sum_{i=1}^3 [(-\lambda f_1 + f_2 + u) S_i \sigma_i + f_1 (\mathbf{S} \times \mathbf{S}_x)_i \sigma_i + \beta (\mathbf{S} \times \mathbf{S}_y)_i \sigma_i] \xi. \end{aligned} \quad (17)$$

where  $\sigma_i$ ,  $i = 1, 2, 3$ , are Pauli matrices,  $f_1$  and  $f_2$  are given by (16), and the spectral parameter satisfies the nonlinear equation (15).

It is interesting to note that the inhomogeneous LME (4) admits the following integrable reductions:

i) When  $\beta = 0$ , it reduces to the inhomogeneous HFE [13]

$$\mathbf{S}_t = \mathbf{S} \times (f_1 \mathbf{S})_{xx} + f_2 \mathbf{S}_x. \quad (18)$$

It should be pointed out that we may also get this reduction by taking  $\partial_x = \partial_y$  and making some simple transformations in (4).

ii) When  $f_1 = 0$  and  $\beta = 1$ , it reduces to the inhomogeneous M-I equation [14],

$$\begin{aligned} \mathbf{S}_t &= \{\mathbf{S} \times \mathbf{S}_y + u \mathbf{S}\}_x + f_2 \mathbf{S}_x, \\ u_x &= -\mathbf{S} \cdot (\mathbf{S}_x \times \mathbf{S}_y). \end{aligned} \quad (19)$$

### 3 L-equivalent and gauge equivalent counterpart

In order to find the L-equivalent counterpart (about our conditional notations, see e.g. Refs. [9-10]) of the inhomogeneous LME (4), we now consider the space curve in 3-dimensional space  $E^3$  with the arclength  $x$ ,

the curvature  $\kappa$  and the torsion  $\tau$ . Let  $\mathbf{e}_1, \mathbf{e}_2$  and  $\mathbf{e}_3$  are the unit tangent, normal and binormal vectors of a curve, respectively. Then we have the following equations [6,8],

$$\mathbf{e}_{jx} = \mathbf{A} \times \mathbf{e}_j, \quad \mathbf{e}_{jy} = \mathbf{B} \times \mathbf{e}_j, \quad \mathbf{e}_{jt} = \mathbf{C} \times \mathbf{e}_j, \quad (20)$$

where

$$\mathbf{A} = \tau \mathbf{e}_1 + k \mathbf{e}_3 = (\tau, 0, k), \mathbf{B} = \gamma_1 \mathbf{e}_1 + \gamma_2 \mathbf{e}_2 + \gamma_3 \mathbf{e}_3 = (\gamma_1, \gamma_2, \gamma_3), \mathbf{C} = \omega_1 \mathbf{e}_1 + \omega_2 \mathbf{e}_2 + \omega_3 \mathbf{e}_3 = (\omega_1, \omega_2, \omega_3), \quad (21)$$

and

$$\mathbf{A}_t - \mathbf{C}_x + \mathbf{A} \times \mathbf{C} = 0, \quad \mathbf{A}_y - \mathbf{B}_x + \mathbf{A} \times \mathbf{B} = 0, \quad \mathbf{B}_t - \mathbf{C}_y + \mathbf{B} \times \mathbf{C} = 0. \quad (22)$$

By taking  $\mathbf{S} \equiv \mathbf{e}_1$  and using (21), we have

$$\begin{aligned} \mathbf{B} &= (\gamma_1, \gamma_2, \gamma_3) = \left( \frac{1}{\beta} u + \partial_x^{-1} \tau_y, \frac{1}{\beta \kappa} u_x, \partial_x^{-1} (\kappa_y - \frac{\tau}{\beta \kappa} u_x) \right), \\ \mathbf{C} &= (\omega_1, \omega_2, \omega_3) = \left( \frac{f_1}{\kappa} \kappa_{xx} + \frac{\beta}{\kappa} \kappa_{xy} - f_1 \tau^2 + f_2 \tau - \beta \tau \partial_x^{-1} \tau_y, -f_1 \kappa_x - \beta \kappa_y, f_2 \kappa - f_1 \kappa \tau - \beta \kappa \partial_x^{-1} \tau_y \right). \end{aligned} \quad (23)$$

Substituting (23) into (22), we obtain the following equations for curvature and torsion

$$\begin{aligned} \kappa_t &= (f_2 \kappa - f_1 \kappa \tau - \beta \kappa \partial_x^{-1} \tau_y)_x - f_1 \kappa_x \tau - \beta \kappa_y \tau, \\ \tau_t &= \left[ \frac{f_1}{\kappa} \kappa_{xx} + \frac{\beta}{\kappa} \kappa_{xy} - f_1 \tau^2 + f_2 \tau - \beta \tau \partial_x^{-1} \tau_y \right]_x - f_1 \kappa \kappa_x - \beta \kappa \kappa_y. \end{aligned} \quad (24)$$

In terms of the complex function

$$q = \frac{\kappa}{2} e^{-i \partial_x^{-1} \tau}, \quad (25)$$

we may rewritten (24) as

$$\begin{aligned} i q_t - (f_1 q)_{xx} - \beta q_{xy} - i (f_2 q)_x - v q &= 0, \\ i p_t + (f_1 p)_{xx} + \beta p_{xy} - i (f_2 p)_x + v p &= 0, \\ v_x &= 2[(f_1 p q)_x + \beta (p q)_y], \end{aligned} \quad (26)$$

that is (2+1)-dimensional NLSE. Here  $p = \varepsilon \bar{q}$  and  $\varepsilon = 1$  ( $\varepsilon = -1$ ) corresponds to the focusing (defocusing) case. The Lax representation of (26) is given by

$$\begin{aligned} \Phi_x &= U \Phi, \\ \Phi_t &= \lambda \beta \Phi_y + V \Phi, \end{aligned} \quad (27)$$

where

$$U = \frac{i\lambda}{2} + G, \quad G = \begin{pmatrix} 0 & q \\ p & 0 \end{pmatrix}, \quad V = \frac{i\lambda^2}{2} f_1 \sigma_3 + \frac{i\lambda}{2} f_2 \sigma_3 + \lambda V_1 + V_0, \quad (28)$$

in which

$$V_1 = f_1 G, \quad V_0 = \begin{pmatrix} i f_1 p q + i \beta \partial_x^{-1} (p q)_y & f_2 q - i \beta q_y - i (f_1 q)_x \\ f_2 p + i \beta p_y + i (f_1 p)_x & -[i f_1 p q + i \beta \partial_x^{-1} (p q)_y] \end{pmatrix}, \quad (29)$$

and the spectral parameter  $\lambda(y, t)$  satisfies the equation (15). Eq. (26) also admits two reductions:

i) When  $\beta = 0$ , it reduces to the inhomogeneous (1+1)-dimensional NLSE

$$\begin{aligned} i q_t - (f_1 q)_{xx} - i (f_2 q)_x - v q &= 0, \\ i p_t + (f_1 p)_{xx} - i (f_2 p)_x + v p &= 0, \\ v_x &= 2(f_1 p q)_x, \end{aligned} \quad (30)$$

ii) When  $f_1 = 0$  and  $\beta = 1$ , it reduces to the inhomogeneous (2+1)-dimensional NLSE

$$\begin{aligned} i q_t - q_{xy} - i (f_2 q)_x - v q &= 0, \\ i p_t + p_{xy} - i (f_2 p)_x + v p &= 0, \\ v_x &= 2(p q)_y. \end{aligned} \quad (31)$$

Note that the gauge equivalence between (4) and (26) can also be established by taking the following transformation between the solutions of systems (17) and (27)

$$\Psi = g^{-1} \Phi, \quad (32)$$

where  $g = \Phi|_{\lambda=0}$ .

## 4 Conclusion and remarks

In this paper, we have constructed an integrable inhomogeneous extension of (1) and given the corresponding L-equivalent counterpart which is the (2+1)-dimensional generalized NLSE. It should be noted that the homogeneous HFE admits several integrable extensions in 2+1 dimensions, such as

1<sup>0</sup>. The Myrzakulov-VIII (M-VIII) equation [5]

$$\begin{aligned} \mathbf{S}_t &= \mathbf{S} \times \mathbf{S}_{xx} + u \mathbf{S}_x, \\ u_y &= \mathbf{S} \cdot (\mathbf{S}_x \times \mathbf{S}_y), \end{aligned} \quad (33)$$

2<sup>0</sup>. The Ishimori equation [4]

$$\begin{aligned} \mathbf{S}_t &= \mathbf{S} \times (\mathbf{S}_{xx} + \alpha^2 \mathbf{S}_{yy}) + u_x \mathbf{S}_y + u_y \mathbf{S}_x, \\ u_{xx} - \alpha^2 u_{yy} &= -2\alpha^2 \mathbf{S} \cdot (\mathbf{S}_x \times \mathbf{S}_y), \end{aligned} \quad (34)$$

3<sup>0</sup>. The Myrzakulov-IX (M-IX) equation [5]

$$\begin{aligned}\mathbf{S}_t &= \mathbf{S} \times M_1 \mathbf{S} + A_1 \mathbf{S}_y + A_2 \mathbf{S}_x, \\ M_2 u &= 2\alpha^2 \mathbf{S} \cdot (\mathbf{S}_x \times \mathbf{S}_y).\end{aligned}\tag{35}$$

Whether all of these equations admit inhomogeneous extensions is still a question for the future.

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